

Emotion

An Exploratory Analysis of Emotion Dynamics Between Mothers and Adolescents During Conflict Discussions

Alexandra Main, Alexandra Paxton, and Rick Dale

Online First Publication, May 5, 2016. <http://dx.doi.org/10.1037/emo0000180>

CITATION

Main, A., Paxton, A., & Dale, R. (2016, May 5). An Exploratory Analysis of Emotion Dynamics Between Mothers and Adolescents During Conflict Discussions. *Emotion*. Advance online publication. <http://dx.doi.org/10.1037/emo0000180>

An Exploratory Analysis of Emotion Dynamics Between Mothers and Adolescents During Conflict Discussions

Alexandra Main
University of California, Merced

Alexandra Paxton
University of California, Berkeley

Rick Dale
University of California, Merced

Dynamic patterns of influence between parents and children have long been considered key to understanding family relationships. Despite this, most observational research on emotion in parent–child interactions examines global behaviors at the expense of exploring moment-to-moment fluctuations in emotions that are important for relational outcomes. Using recurrence quantification analysis (RQA) and growth curve analysis, this investigation explored emotion dynamics during parent–adolescent conflict interactions, focusing not only on concurrently shared emotional states but also on time-lagged synchrony of parents’ and adolescents’ emotions relative to one another. Mother–adolescent dyads engaged in a 10-min conflict discussion and reported on their satisfaction with the process and outcome of the discussion. Emotions were coded using the Specific Affect Coding System (SPAFF) and were collapsed into the following categories: negativity, positivity, and validation/interest. RQA and growth curve analyses revealed that negative and positive emotions were characterized by a concurrently synchronous pattern across all dyads, with the highest recurrence rates occurring around simultaneity. However, lower levels of concurrent synchrony of negative emotions were associated with higher discussion satisfaction. We also found that patterns of negativity differed with age: Mothers led negativity in dyads with younger adolescents, and adolescents led negativity in dyads with older adolescents. In contrast to negative and positive emotions, validation/interest showed the time-lagged pattern characteristic of turn-taking, and more highly satisfied dyads showed stronger patterns of time-lagged coordination in validation/interest. Our findings underscore the dynamic nature of emotions in parent–adolescent interactions and highlight the important contributions of these moment-to-moment dynamics toward overall interaction quality.

Keywords: adolescence, conflict, emotion dynamics, recurrence quantification analysis, synchrony

Supplemental materials: <http://dx.doi.org/10.1037/emo0000180.supp>

Dynamic, transactional patterns of influence between parents and children have long been acknowledged as key to understanding family relationships (Granic, 2008; Maccoby & Martin, 1983). Studying dynamic aspects of emotion in interpersonal interactions is gaining increased attention as researchers move beyond purely intrapsychic approaches to studying emotion and empha-

size the role of mutual influence of social partners’ emotions on one another during interpersonal interactions (e.g., Butler, 2015; Campos, Walle, Dahl, & Main, 2011; Netzer, Van Kleef, & Tamir, 2015; Zaki & Williams, 2013). Adolescence is a unique developmental period in which to study emotion dynamics because parent–child relationships undergo transformations that may increase conflict and negative emotions (Laursen & Collins, 2004). Microlevel processes during parent–adolescent interactions are important for advancing understanding of how to prevent problem behavior during this often high-risk developmental period (Granic, 2008).

Recent advances in research on emotion dynamics during parent–adolescent interactions include analyzing flexibility (e.g., Connell, Hughes-Scalise, Klostermann, & Azem, 2011; Hollenstein & Lewis, 2006; Lougheed & Hollenstein, 2016), sequential emotion contingencies (Lichtwarck-Aschoff, Kunnen, & van Geert, 2010), and coregulation (Lougheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015) during these interactions from a dynamic systems perspective. Despite these advances, most observational research on parent–adolescent interactions focuses on the unidirectional influences of parents’ behavior on adolescent outcomes and centers on either concur-

Alexandra Main, Department of Psychological Sciences, University of California, Merced; Alexandra Paxton, Institute of Cognitive and Brain Sciences, University of California, Berkeley; Rick Dale, Department of Cognitive and Information Sciences, University of California, Merced.

This research was supported by grants from the Jacob K. Javits Foundation, the University of California, Berkeley Institute of Human Development, the Greater Good Science Center, and the Psi Chi National Honors Society awarded to Alexandra Main. We thank Michelle Russell and Emma Netland for assisting in data coding. We are also grateful to the families that participated in or contributed to the study.

Correspondence concerning this article should be addressed to Alexandra Main, Department of Psychological Sciences, 5200 North Lake Road, University of California, Merced, CA 95343. E-mail: amain@ucmerced.edu

rently displayed emotion or sequential influence of one family member's emotions on the other's.

The present study expands this line of research by exploring how dynamic, temporal aspects of different emotional states (i.e., negativity, positivity, and validation/interest) displayed during mother–adolescent conflict discussions relate to perceptions of how well conflicts were managed (i.e., dyadic satisfaction with the outcome and process of a conflict discussion) and how such patterns might vary across early and late adolescence. Uniquely, we focused not only on concurrently shared emotional states but also on the time-lagged synchronization of mothers' and adolescents' emotions *relative to one another*. Specifically, we used recurrence quantification analysis (RQA) and growth curve modeling to explore the characteristic lag time for mothers and adolescents to match one another's emotional states across (a) all dyads regardless of discussion satisfaction, (b) level of dyadic satisfaction with the process and outcome of a conflict discussion, and (c) adolescent age.

Interpersonal Emotion Dynamics

While theoretical and empirical approaches to emotion still predominantly focus on the individual's emotional experience in largely solitary contexts, interpersonal approaches to studying emotion that emphasize patterns of influence between individuals are gaining increased attention (cf. Butler, 2015; Campos et al., 2011; Kuppens, 2015; Zaki & Williams, 2013). In this framework, the emotions of one person are inherently linked with those of others. Understanding the temporal aspects of these linkages is important for appreciating how emotions play out over time and influence interactional outcomes. Analyses of real-time temporal dynamics of emotion between social partners contribute to theoretical and empirical perspectives on emotion above and beyond aggregate measures (Butler, 2011) and inform more targeted interventions aimed at preventing negative emotional patterns from harming relationships over time (Granic, 2005).

Emotion dynamics between individuals have been described a multitude of ways in recent years, including covariation, coordination, linkage, reciprocity, transmission, contagion, synchrony, and coregulation (for review, see Butler, 2011). We recognize that these terms have a host of different uses across areas of psychology and cognitive science with little consensus in their definitions (cf. Butler, 2011; Delaherche et al., 2012; Paxton & Dale, 2013). Drawing from the broader psychology and cognitive science literature on coordination and synchrony, in the current paper we conceptualize coordinated actions and emotions as primarily taking one of two stable patterns: *concurrent synchrony* and *time-lagged synchrony*. We operationally define each below.

A key concern about the dynamic structure of emotion during interpersonal interaction is the nature of the synchrony that holds between persons. Synchrony has been defined many ways across multiple disciplines, but in the context of interpersonal emotion dynamics, synchrony refers to covariation of social partners' emotions in unison (Butler, 2015). *Concurrent synchrony* refers to linkages between interactional partners' emotions in their current state (i.e., lag time of zero), while *time-lagged synchrony* refers to correspondence between partners' emotions at prior and subsequent time lags (see Butler, 2011, for a review). Interactions are characterized by a high degree of concurrent synchrony if both

members of the dyad are more likely to be in the same emotional state at the same time than across all other possible time lags. Conversely, emotions between individuals are characterized by time-lagged synchrony if a change in one person's emotional state precedes or follows a change in the other's state (Feldman, 2003).

Previous research has shown that concurrent synchrony of positive emotions (i.e., simultaneously displayed positive emotions) is associated with positive outcomes in families with infants and young children, including increased child self-regulation, cognitive abilities, and empathy (Feldman, 2007), lower maternal depression (Field, Healy, Goldstein, & Guthertz, 1990), and secure attachment (Lindsey & Caldera, 2014). Conversely, concurrent negative emotional synchrony has been associated with conduct problems (Patterson, 1982) and poor emotion regulation (Cole, Teti, & Zahn-Waxler, 2003). From a dynamic systems perspective, simultaneous displays of negativity reflect a tendency to get "stuck" in negative states, especially in problematic relationships (Hollenstein & Lewis, 2006). These outcomes associated with concurrent emotional synchrony no doubt emerge over long histories of interaction patterns between dyads (Granic, 2005). Less research has examined whether such patterns are important for immediate interactive outcomes.

Whereas concurrent synchrony refers to emotion linkages between individuals in their current state, time-lagged synchrony reflects a pattern in which a change in one partner's emotion is matched by a change in the other's emotion at an earlier or later point in the interaction (Feldman, 2003). Time-lagged synchrony may reflect an out-of-phase pattern and can be indicative of a turn-taking dynamic (see Butner, Berg, Baucom, & Wiebe, 2014). For example, one would observe a turn-taking pattern in negative emotions if one person's negativity were less likely to occur at the same time as the partner's negativity and showed a characteristic lag from their partner's negativity. Previous research has found that time-lagged synchrony characterized by a turn-taking pattern varies across gender in romantic couples (Randall, Post, Reed, & Butler, 2013) and as a function of shared visual (Richardson, Marsh, & Schmidt, 2005) and emotional experiences (Vallacher, Nowak, & Zochowski, 2005).

Analyses of time series, such as cross-correlation, have proven useful for analyzing concurrent and time-lagged synchrony between emotions of social partners (see Butler, 2011). Such analyses reveal the likelihood that one individual is in a given state across all possible time lags from the partner's designated state. Time series analyses focusing on time-lagged synchrony can also provide information about which person is leading changes in such states. This can inform researchers as to who is "driving" the interaction (Feldman, 2006). For example, studies have shown that mothers tend to lead changes in emotion during mother–infant interactions (Feldman, 2006), and same-gender mother–infant dyads show more mutual covariation (i.e., less of a leader–follower pattern) than mixed-gender dyads (Feldman, 2003). Given important developmental transitions in autonomy a shift from more vertical to horizontal relationships between parents and children during adolescence (Laursen & Collins, 2004), the present study explored leader/follower patterns of emotion dynamics in the context of parent–adolescent interactions.

Most research on emotion dynamics has examined negativity and positivity broadly, but specific emotional communication behaviors are hypothesized to serve distinct functions during social

interaction (Coan & Gottman, 2007). Most notably, parental validation (i.e., conveying an understanding of another's thoughts and feelings) and interest (i.e., curiosity about another's point of view and feelings) displayed during parent–adolescent interactions are important for child and adolescent adjustment (Allen, Hauser, Bell, & O'Connor, 1994; Loughheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015) and effective conflict resolution (Halpern, 2007). Validation and interest may inhibit destructive and promote constructive behaviors during arguments and downregulate negative emotion (Halpern, 2007).

Although validation and interest could be considered broadly positive in valence, there is reason to expect that validation and interest may be characterized by different patterns of synchrony during interpersonal interactions compared with other positive emotions such as humor or affection. While humor and affection are likely to be displayed simultaneously (i.e., with concurrent synchrony) given their functions to communicate mutual affiliation (Coan & Gottman, 2007), validation and interest function to convey an understanding or a curiosity about *the other* (Coan & Gottman, 2007; Halpern, 2007). Thus, although both interactive partners may show validation and interest during an interaction, they are less likely to engage in these behaviors simultaneously and may be more likely to exhibit a turn-taking pattern for these behaviors. For example, an interaction may begin with the parent exhibiting interest while listening to the adolescent's point of view. Once the adolescent has finished speaking, the parent may validate the adolescent's point of view before offering a different perspective. The adolescent may then begin to display interest while listening to the parent's point of view. Such a pattern would reflect mutual understanding and may be especially important during conflict, because such turn-taking involves a willingness to understand one another's point of view (Halpern, 2007).

Emotion in Parent–Adolescent Interactions

The majority of research on emotion dynamics has been conducted with infants, young children, and adults (see Butler, 2011, for a review). Considerably fewer studies have examined adolescents and their parents. Adolescence is a developmental period when parent–child relationships undergo transformations that can increase conflict and negative emotion (Laursen & Collins, 2004). Negative emotion during conflict tends to increase from early to mid-adolescence, with a decline in late adolescence (Laursen, Coy, & Collins, 1998). Adolescents whose interactions with their parents are characterized by negative emotion are more likely to have poor adjustment, including externalizing problems and less adaptive emotion regulation (Loughheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015; Moed et al., 2015; Patterson, 1982).

General features of parent–adolescent relationships may be captured by self-report or by aggregating codes of behavior during interactions, but a dynamical approach provides a window into finer-grained patterns of emotional communication that can become characteristic of a relationship over time (Granic, 2005; Loughheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015). Studies that have analyzed interpersonal emotion dynamics in parent–adolescent relationships often examine dependent variables such as mutual positivity (e.g., Connell et al., 2011) or flexibility (e.g., Hollenstein & Lewis, 2006). However, it is equally important to examine the functional outcomes of interpersonal emotion dy-

namics (Butler, 2011). Interpersonal conflict management is one such functional outcome with strong implications for the quality of parent–adolescent relationships over time (Granic, 2005; Laursen & Collins, 2004).

Tracking these dynamics requires the adaptation of existing—or development of entirely new—statistical techniques. Other studies that have examined emotion dynamics in parent–adolescent interactions have utilized a variety of techniques, including state-space grids (e.g., Hollenstein & Lewis, 2006; Lichtwarck-Aschoff et al., 2009), sequential analyses (e.g., Lichtwarck-Aschoff et al., 2010), and event history analysis (e.g., Loughheed, Craig, et al., 2015; Loughheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015). Such methods have been crucial for understanding temporal dynamics of emotion in parent–adolescent relationships. Similar to the aforementioned studies, we utilized second-by-second categorical emotion codes to explore such temporal dynamics, creating a discrete event series for dynamic analysis.

In the present study, we contribute to the ongoing agenda of research to unveil emotion dynamics in real-time parent–adolescent interactions by using *recurrence quantification analysis (RQA)*, which can quantify the timing of the relative occurrence of particular emotions within and across individuals (Dale, Warlaimont, & Richardson, 2011). We leveraged this technique to ask exploratory questions about the emotional synchrony between adolescents and parents. What kinds of emotional synchrony (concurrent and/or time-lagged), if any, occur during parent–adolescent interactions, how might such synchrony relate to discussion satisfaction, and how might it vary across adolescent age? Below we briefly introduce RQA and showcase how it works with a hypothetical analysis.

Recurrence Quantification Analysis

RQA (Webber & Zbilut, 2005) is a nonlinear framework for analyzing the dynamic structure in a time series. This descriptive statistical technique allows researchers to uncover the fine-grained temporal structure of interpersonal systems (Dale et al., 2011). RQA is typically applied in one of two ways. First, the physical and biological sciences commonly use RQA to describe the dynamic structure of one continuous time series (see Marwan, 2008, for a review) and to analyze two different time series (Zbilut, Giuliani, & Webber, 1998). Second, RQA can be used to study *categorical* time series (Lichtwarck-Aschoff, Hasselman, Cox, Pepler, & Granic, 2012), allowing a researcher to extract a sequence of categories (e.g., emotion states) across all possible time delays within an interaction (e.g., Dale & Spivey, 2006; Dale et al., 2011; for a recent review see Fusaroli, Konvalinka, & Wallot, 2014).

There are a number of benefits to using RQA over standard cross-correlation. First, cross-correlation is based on covariation of magnitudes and has a “true 0,” which is factored into calculation of coefficients. However, when examining emotion in interpersonal interactions, not expressing an emotion is not a “true 0.” In other words, we do not wish to treat *shared absence* of a target emotion in two people as a kind of synchrony of that emotion. RQA allows us to factor these events out of our analysis. Second, RQA results can be expressed transparently in terms of percentage of recurrence events. This makes results somewhat easier to interpret than those of cross-correlation. Third, the RQA approach is

anchored to an emerging dynamic approach that is motivated more by “co-visitation” of states (i.e., recurrences) than “co-variation” of states (i.e., variances and covariances). Thus, RQA allows for analysis of behavioral streams of nominal states (such as emotion states) while simultaneously revealing nonlinear patterns in coupled systems (Dale et al., 2011).

There are multiple ways that RQA can be used to explore categorical time series. One study that used categorical RQA to examine emotion in parent–child interactions cleverly collapsed dyadic codes into one “dyad time series” and subjected it to what is sometimes termed *auto-recurrence quantification analysis* (Lichtwarck-Aschoff et al., 2012). Here, we analyzed dyads’ emotion dynamics by preserving mothers’ and adolescents’ paired time series as separate by using *cross-recurrence quantification analysis*. We took the event series for the parent and the adolescent separately and quantified when different emotion states co-occurred in relative time (i.e., demonstrated recurrence). For example, if parents and adolescents both showed positive emotions at the same time, RQA would characterize the emotional synchrony for positive emotions as concurrent. On the other hand, if mothers reliably exhibited positive emotion at a characteristic time lag *after* adolescents’ displays of positive emotion, then RQA would reveal a pattern of time-lagged synchrony for positive emotions.

Hypothetical Example of RQA

Figure 1 shows how RQA can be applied to the analysis of two time series across several versions of potential relationships between two emotional event time series. For each panel, the left column represents the emotion time series for Person 1 and Person 2. RQA first builds a recurrence plot (see center column of Figure 1). Here, Person 1’s event series is on the x axis, and Person 2’s event series is on the y axis, generating a cross-recurrence plot (CRP). Points on the plot represent when the systems (in this case, the mother and the adolescent in each dyad) show *recurrent* (i.e., the same) emotion states. This provides a visualization of the recurring patterns of emotions across the interaction.

From this plot, we are able to extract rich information about the emotion dynamics. If two people are mainly concurrent, most points will occur near the diagonal of the plot (lag = 0), indicating they are generally in the same emotional state at the same time. This pattern is represented in Panel A. If one person’s emotions are lagged behind the other’s, the CRP would show clusters of points shifted off-center (Panel B). The rightmost column shows a *diagonal recurrence profile (DRP)* quantifying how the points are clustering near the diagonal.

DRPs can be understood as similar to a cross-correlation function (Dale et al., 2011) that reveal the relationship between two time series (although with the added capabilities described earlier). DRPs show the *recurrence rate* (percentage of recurrence) along the y axis and the relative time lag on the x axis. This provides a visualization of the concurrent or time-lagged recurrence of emotion states between parents and adolescents. For example, in Panel A, the DRP is a virtually unimodal distribution centered at a lag of 0 (i.e., the main diagonal on the CRP), reflecting a dyad that tends to display the same emotion at the same time. Conversely, Panel B presents a bimodal coupling function that indicates a turn-taking pattern. Leader/follower dynamics are represented in Panel C. If leader/follower dynamics are consistent across the interaction, the

DRP will be shifted either right or left (depending on who is leading). Finally, if Person 1 and Person 2’s emotions show no relationship in time, then their DRP will be approximately flat (Panel D).

RQA and Growth Curve Analyses

We complemented the descriptive RQA framework with inferential statistics to test whether DRPs show significant concurrent and time-lagged synchrony patterns across different emotion types. Specifically, we applied growth curve analyses to DRPs to test both quadratic and linear lag terms to predict recurrence rate (Mirman et al., 2008). Growth curve analysis of DRPs is similar to lag sequential analysis, although lag sequential analysis is most commonly applied to only a small number of time lags. In a growth curve framework, one can statistically test the DRPs across *all* possible lags, contrasting each time point with every other time point in the series (see Bakeman & Quera, 2011, for a description and comparison).

When combining RQA and growth curve analyses, the outcome variable is the recurrence rate, which is predicted by lag and any other variables of interest (e.g., experimental condition). Both linear and quadratic forms of the lag term can be included as predictors of recurrence rate to find evidence of leader/follower patterns (using the linear term) and to uncover concurrent or time-lagged synchrony apparent in the shape of the recurrence profile (using the quadratic term). Interactions between lag type and the independent variable(s) of interest can highlight whether the leader/follower dynamics and type of synchrony differ across levels of the variable of interest.

Previous research using RQA in the context of dyadic interactions has examined the coordination of movement patterns (e.g., Abney, Paxton, Dale, & Kello, 2015), heart rate (e.g., Konvalinka et al., 2011), interactive timing of language signals between mothers and infants (e.g., Warlaumont, Richards, Gilkerson, & Oller, 2014), and nonlinear change processes during parent–child interactions over the course of treatment for families with an aggressive child (Lichtwarck-Aschoff et al., 2012). However, to our knowledge, RQA has not been used to analyze emotion dynamics in the context of parent–adolescent conflict in a typically developing sample.

The Present Study

The present study used RQA and growth curve analyses to explore observed emotion dynamics between mothers and adolescents during conflict discussions. Specifically, we examined (a) patterns of mother–adolescent emotion dynamics of negative, positive, and validation/interest states, (b) relations between emotion dynamics and reported satisfaction with the process and outcome of a conflict discussion, and (c) potential age differences in emotion dynamics between younger (13- to 14-year-olds) and older (17- to 18-year-olds) adolescents and their mothers.

Though the goal of the present study is primarily to explore patterns of emotion dynamics during parent–adolescent conflict, the previous research cited above provided a framework for some specific hypotheses for the analyses in the present study. First, we hypothesized that mothers’ negative emotions would co-occur in a relatively concurrently synchronous pattern (i.e., occur simultane-

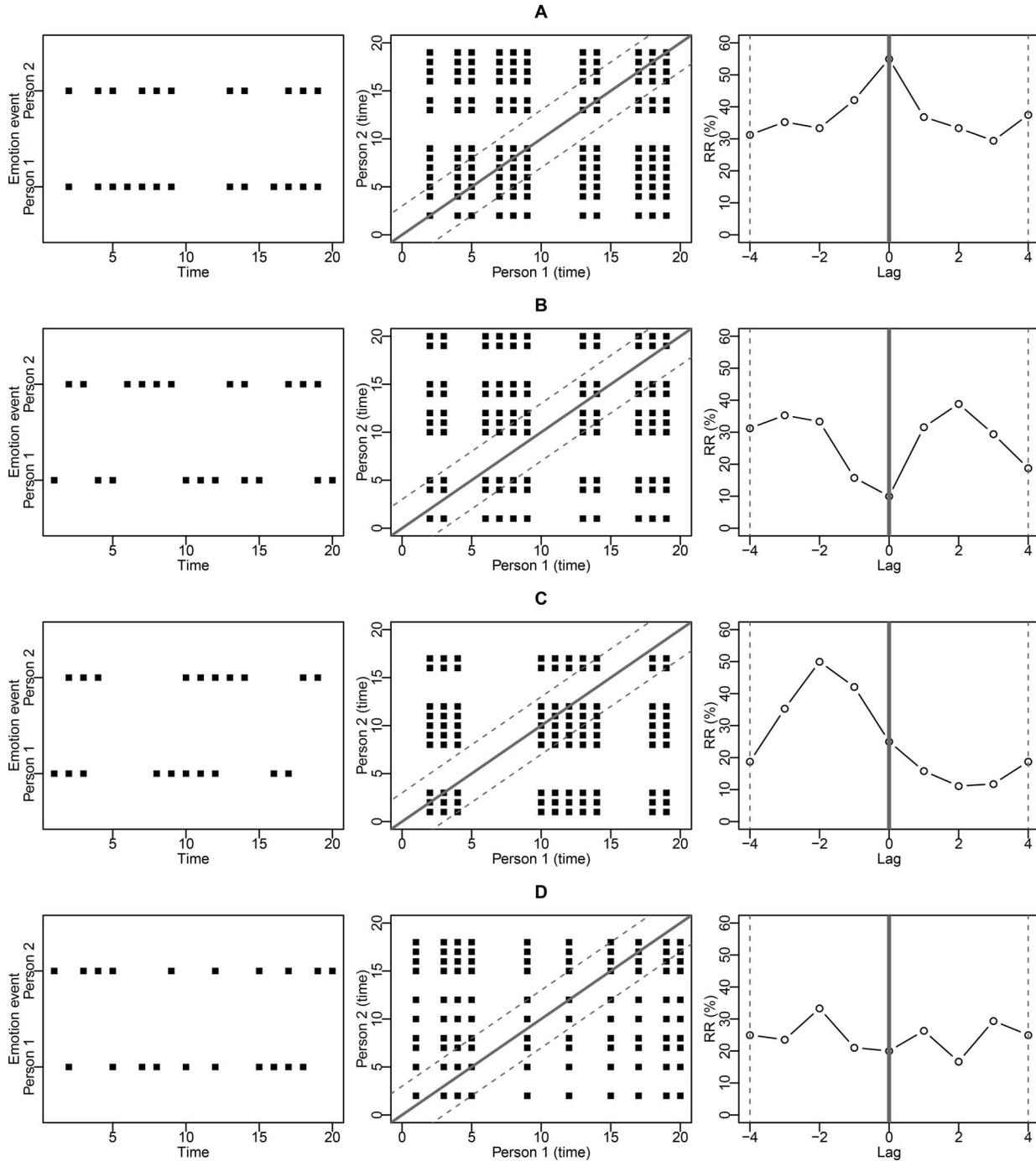


Figure 1. Illustration of different types of potential synchrony for two interacting individuals (Person 1 and Person 2). For each panel, the left column represents each person's emotion time series. The center column represents a cross-recurrence plot (CRP) of each time series. Points on this plot represent relative time points when the two people are in matching states. Calculating percentage recurrence (RR) along the diagonals of this plot yields a main line (thick gray line) representing the absolute "line of synchronization" (lag = 0). The rightmost column shows a diagonal recurrence profile (DRP). In the top panel (A), Person 1 and Person 2 show an emotional event series at relatively concurrent synchrony (matching states). In (B), Person 1 and Person 2 are in time-lagged synchrony, showing a turn-taking pattern. In (C), one person tends to lead the other (Person 1 leads Person 2). Finally, the bottom row (D) represents two people who do not show relative synchrony, resulting in a flat DRP. In order to assess significance of these patterns, we compute DRPs across all dyads and use growth-curve modeling, as described in the paper.

ously) with adolescents' negative emotions, and we expected to find a similar (i.e., concurrently synchronous) pattern for positive emotions. We also expected validation/interest states to reflect a turn-taking pattern or time-lagged synchrony, in which individuals would be *less* likely to exhibit these behaviors simultaneously. Second, we hypothesized that dyads with higher discussion satisfaction would show less concurrent synchrony of negative emotions and more concurrent synchrony of positive emotions. We also expected that dyads with higher discussion satisfaction would show greater levels of time-lagged synchrony in validation/interest states, reflecting a turn-taking pattern. Third, given the increase in negative emotions in parent-adolescent interactions in early adolescence and their decline in late adolescence, we expected greater concurrent synchrony of negative emotion among dyads with younger adolescents compared with older adolescents. However, because previous research has primarily focused on changes in negative emotion in parent-adolescent interactions over the course of adolescence, we did not have any specific predictions about emotion synchrony across adolescent age for positivity or validation/interest.

Method

Participants

Participants included 50 adolescents (30 female; M age = 14.84 years, $SD = 1.99$) and their mothers, recruited from local schools and communities in the San Francisco Bay Area. To test age-related differences in emotion dynamics, adolescents were recruited from the following age ranges: 13 to 14 years old ($N = 29$) and 17 to 18 years old ($N = 21$). One dyad from the younger age group was excluded from the analyses because of an error in researcher instruction during the interaction portion of the task, resulting in an analysis of 28 mother-adolescent dyads for the younger group.

Participants were recruited using a variety of methods, including through schools, teen afterschool programs, parenting groups, and parent/teen newsletters. The investigation was described as a research study on how mothers and teens talk about conflict. Eligibility was based on the following criteria: (a) the adolescent was 13 to 14 or 17 to 18 years old at the time of testing, (b) the adolescent lived at least five days a week with the participating mother, and (c) both mother and adolescent were able to understand and speak English. Mothers were given a \$20 check, and adolescents were given a \$20 gift card for participating.

The racial/ethnic breakdown of the dyads was as follows: 62% non-Hispanic white, 16% non-Hispanic black, 10% Asian/Pacific Islander, 4% Hispanic, and 8% other. Maternal education ranged from a high school degree to an advanced graduate degree, with the median highest degree obtained being a bachelor's degree (36.0%). Families' annual income ranged from less than \$25,000 per year to more than \$150,000 per year, with the average family income in the \$81,000 to \$100,000 range.

Procedure

Each mother-adolescent dyad participated in a 1.5-hr laboratory visit. During this time, participants completed various survey measures and had a 10-min discussion of a topic of frequent and

intense conflict in their relationship. After obtaining written consent from mothers and assent or consent from adolescents, mothers and adolescents separately completed an issues checklist to help identify the topic of the conflict discussion. The lead author then assisted the dyad in deciding on a topic from the checklist to discuss and interviewed the dyads for five minutes about the chosen topic to help them focus on the key area(s) of disagreement.

Each dyad discussed their identified topic for 10 minutes without a researcher present. Mothers and adolescents sat across from one another (approximately one meter apart) at a small table. Two visible video cameras (one facing each participant) captured the participants from the top of the head to the midchest. The conversations were monitored via one-way mirror. After 10 min had elapsed, the lead author reentered the room and signaled the end of the discussion. Following the discussion, mothers and adolescents separately rated their satisfaction with the discussion.

Materials

Identification of conflict topics. Mothers and adolescents separately completed a modified version of the Issues Checklist (Prinz, Foster, Kent, & O'Leary, 1979), a well-accepted tool in parent-adolescent conflict research. This checklist identifies common sources of conflict between parents and adolescents. For each issue, mothers and adolescents separately rated (a) whether a given topic was an issue in their relationship, and (b) if yes, how upsetting was the issue on a scale of 1 (*slightly upsetting*) to 5 (*very upsetting*). The topic of "telephone calls" was changed to "telephone calls/texting," and a "social media" topic was added to modernize the questionnaire. Mothers and adolescents independently identified the two topics that were most upsetting to them.

Behavioral coding. The Specific Affect Coding System (SPAFF Version 4.0; Coan & Gottman, 2007) was used to code mother and adolescent emotions displayed during the conflict discussions. The SPAFF is divided into positive, negative, and neutral codes, with specific emotions within each broad dimension. Uniquely, the SPAFF considers a gestalt of verbal content, voice tone, context, facial expression, gesture, and body movement cues in determining the presence of each emotion, meaning codes could be verbal, nonverbal, or both. The SPAFF considers the stream of behavior as continuous, allowing for codes to be assigned at any time. Mothers and adolescents were coded separately, resulting in two synchronized streams of behavior. SPAFF codes were assigned in a mutually exclusive and exhaustive manner in the present investigation, meaning that only one code was applied at any given unit of time. Timed event-based coding was used, meaning that coders indicated the onset and offset time of each emotion (see Bakeman & Quera, 2011).

SPAFF codes were recorded using Mangold INTERACT (Version 14). The lead author trained two undergraduate research assistants to reach 75% agreement on training videos across all codes prior to the start of coding. Reliability was based on second-by-second concordance of observers' codes throughout the 10-min interaction. All interactions were coded by the lead author and the two undergraduate coders, with the former serving as the "gold standard" to which other observers' codes were compared, as recommended by Coan and Gottman (2007). Weekly calibration checks and discussions were held to minimize coder drift. Reliability was checked for each dyad, and a minimum of 75%

agreement across all SPAFF codes was required in order for the data to be included in the final analyses. Cohen's kappa was used to calculate interrater reliability. The average kappa across all codes was .77 and .75 (range = .62 to .88) for mother and adolescent codes, respectively. We focused on three broad emotion categories: negativity, positivity, and validation/interest.¹ Each category was created by collapsing across all SPAFF codes in that category.

Discussion satisfaction. Immediately after the conflict discussion, mothers and adolescents separately rated the process and outcome of the discussion with two items: "How satisfied were you with the outcome of the discussion?" and "How satisfied were you with the way the discussion went?" on a scale of 1 (*not at all*) to 5 (*completely*). Responses to these two questions were highly correlated within both mothers ($r = .84, p < .001$) and adolescents ($r = .77, p < .001$). We therefore created a single discussion satisfaction score for mothers and adolescents by computing the mean of these two items for each participant, which were moderately correlated within dyads ($r = .62, p < .001$). To quantify discussion satisfaction at the dyadic level, we calculated the mean of the individual satisfaction composite scores for each mother-adolescent dyad ($Mean_{dyadic} = 4.0$).

Analysis Plan

Recurrence quantification analysis (RQA) and growth curve modeling were used to visualize and analyze emotion dynamics between mothers and adolescents during the conflict discussions. Categorical RQA was used because the emotions displayed during the interactions were coded as either present or absent from moment to moment. Emotion time series for mothers and adolescents were coded in 1-s units (i.e., a sampling rate of 1Hz). Thus, we chose a diagonal recurrence window of ± 30 lags (i.e., ± 30 seconds). This allowed us to capture individuals' concurrent, prior, and subsequent emotion in relation to each second of their partner's emotion at either the beginning or end of the duration of their partner's emotion state. The ± 30 -s window is sufficiently large enough to capture emotional fluctuations that are nearly ± 1 SD of the longest duration of any single emotion ($M = 11$ seconds, $SD = 17.06$ seconds), providing a fuller picture of the emotion dynamics within the dyads. We generated three sets of analyses: The first explored emotion dynamics across all dyads, the second examined the relations between emotion dynamics and dyadic discussion satisfaction, and the third tested potential age-related differences in emotion dynamics. All code for these analyses is provided in the online supplemental materials. We also provide the code and a subset of data on GitHub (<http://www.github.com/a-paxton/emotion-dynamics>).

Growth curve models. We used the approach to mixed-effects models described in Mirman (2014) and Barr (2013). All diagonal recurrence profiles (DRPs) were generated with the *crqa* library in R (Coco & Dale, 2014) and analyzed using growth curve analyses (Mirman et al., 2008). In traditional growth curve analysis, the predictor variables are construed in the context of change (or "growth") over time. Here we adapt growth-curve analysis in order to analyze the temporal profile of recurrence around *lag*.

Lag should not be conflated with *time*. *Lag* quantifies the delay between mothers' time series and adolescents' time series during analysis. A lag of zero analyzes time series as they occurred

simultaneously, comparing the mother's state at time t against the adolescent's state at time t . Increasing lag compares distant points in each time series. Lag is interpreted relative to the sampling rate of the time series; because our time series were sampled at 1 Hz, each unit of lag is equal to an additional delay of one second. For example, a lag of 10 would be comparing the mother's emotional state at time $t + 10$ seconds to the adolescent's state at time t .

With growth curve modeling, we utilize the lag term as a predictor of diagonal recurrence rate (*RR*). In this model, *RR* is the outcome variable, and the linear and quadratic lag terms are used to predict *RR*. The linear term quantifies leading/following. For example, the coefficient for the linear term is negative when the mother is leading in the target emotion (e.g., the onset of mother negativity precedes the onset of adolescent negativity) and positive when the adolescent is leading. A linear term near zero (or not significantly different from zero) indicates that there is no imbalance, with no overall leader or follower evident in the patterns. The quadratic term informs us whether there is concurrent synchrony (positive quadratic term) or time-lagged synchrony (negative quadratic term).

In keeping with recommendations by Mirman et al. (2008), polynomial terms are generated orthogonally to allow contributions of linear and quadratic terms to be considered independently. Analyzing the temporal profile around lag permits us to fit the curvilinear model with linear and quadratic terms and carry out statistical controls of the kind described in Mirman (2014).² Our models can therefore be interpreted as second-order polynomial regression models. See Figure 1 for a visualization of these relationships.

Model specifications. The first set of analyses explored emotion dynamics across dyads without reference to discussion satisfaction or adolescent age. We used the *lme4* library (Bates, Maechler, Bolker, & Walker, 2015) in R to build linear mixed effects regression models. We specified mixed models maximally as long as the model converged; using random intercepts (i.e., dyad membership) and nested the lag terms (i.e., linear and quadratic). If the model failed to converge, we removed terms from the subject-level random effect structure until the most complex model that converged was obtained. In single-equation form, the following describes the mixed-effects model implemented (using formu-

¹ We also examined validation and interest in separate models. The patterns of the findings were similar to those in the joint model, but given their conceptual similarity and expected similar temporal coordination, the two codes were collapsed for the final analyses. Because interest was operationalized as a curiosity about the partner in the present study (see Coan & Gottman, 2007), interest and validation were only coded when the individual was validating or showing interest in the *partner's* point of view or feelings. Thus, if an individual were listening in an interested manner to the partner validating his or her own perspective, interest would not be coded. As with all SPAFF codes, validation and interest can take both verbal (e.g., statements of understanding for validation, questions about the partner's point of view for interest) and nonverbal forms (e.g., affirmative head nodding for validation, positive nonverbal attention for interest).

² A helpful note from a reviewer observed the importance of the zero point selected for the quadratic term. We choose a theoretically justified orthogonal polynomial time term for growth-curve analysis (Mirman, 2014), but it may be possible to adjust the peak of the quadratic term to move it either in parent or child directions to explore maximal model fit. Because our project is exploratory, we are simply seeking the presence of these particular dynamic interactive patterns; for this reason, we leave exploration of the quadratic term to future investigation.

lae conventions described in Barr, 2013). For dyad d at time lag t , the following model can estimate the dyad's RR . Recall that we use the first-order (t') and second-order (t'') orthogonal polynomials for each lag t (per Mirman et al., 2008) to account for separate contributions of each.

$$RR_{dt} = B_0 + B_{0d} + (B_1 + B_{1d})t' + (B_2 + B_{2d})t'' + e_{dt} \quad (1)$$

$B_{\{0,1,2\}}$ represents global coefficients ("fixed effects"), and $B_{\{0,1,2\}d}$ represents the dyad-specific intercepts or slopes (e.g., B_{1d} = the first-order growth-curve slope at the dyad level).³

Below is the statistical model for the second set of analyses that specifies discussion satisfaction as an additional parameter. This second model explores whether the temporal patterning that predicts RR interacts with discussion satisfaction with additional terms for the satisfaction fixed factor (c) and the interactions between linear and quadratic lag and this factor.⁴

$$RR_{dt} = B_0 + B_{0d} + (B_1 + B_{1d})t' + (B_2 + B_{2d})t'' + (B_x + B_{xd})ct' + (B_x + B_{xd})c'' + (B_c + B_{cd})c + e_{dt} \quad (2)$$

In the third set of analyses, we examined whether patterns of emotion dynamics differed as a function of adolescent age. Adolescent age replaced discussion satisfaction in the equations described in Eq. 2.

We placed all outcome and predictor variables on a centered standard scale so that coefficients can be interpreted as standard deviations (effect sizes: Keith, 2005). It is important to note that in multilevel models (such as linear mixed effects models with `lmer`), the issues surrounding effect sizes (including the multi-R) do not yet have consensus interpretation. To address this concern, we also report coefficients in the models with the original scale preserved,⁵ with age and discussion satisfaction centered prior to entry in such models.

Setting parameters. In most RQA applications, a number of parameters have to be set for analysis. We chose the simplest combination of parameters, as these have been shown to be adequate for categorical RQA (Dale et al., 2011; Dale & Spivey, 2006; Richardson & Dale, 2005), and we briefly justify this selection here. Previous work has shown that, for categorical time series, a single embedding dimension ($m = 1$), a default lag of 1 ($\tau = 1$), and a radius of 0 ($r = 0$) are sufficient to extract empirically useful RQA measures (Dale & Spivey, 2006). This means that we examined second-by-second instances of emotion states that match precisely between the mother and adolescent. The reader may also consult Webber and Zbilut (2005) for an elegant introduction to these parameters.

Finally, we performed surrogate analyses, in which we conducted RQA between members of different dyads. Here we should see flat (or nonexistent) recurrence between "artificial" dyads (sometimes referred to as "virtual pairs"). This has been used in past studies as a baseline test of recurrence (see Fusaroli, Konvalinka, & Wallot 2014).

Results

Emotion Dynamics Across All Dyads

We first examined emotion synchrony—comparisons of the same emotion state by mothers and adolescents across various

lags—without reference to discussion satisfaction or adolescent age (see Eq. 1). We hypothesized that negativity and positivity would exhibit a relatively concurrently synchronous pattern and validation/interest would show time-lagged synchrony (i.e., a turn-taking pattern). Results for all three emotion types supported our predictions.

Negativity was significantly concurrently synchronous (quadratic lag: $\beta = -.03$, $p = .001$; Figure 2A). The linear term—which represents DRP slope and can uncover leading/following patterns—did not reach statistical significance ($\beta = -.01$, $p = .18$; Figure 2A). This suggests that negativity did not have a clear leader/follower pattern between mothers and adolescents. These results are supported by a one-sample, two-tailed t test of the maximum lag (i.e., lag at which RR between mother and adolescent was greatest), finding that the maximum lag for negativity was not significantly different from 0, $t(39) = -.42$, $p = .68$. Thus, mothers and adolescents tended to exhibit negativity simultaneously.

Positive emotion was also concurrently synchronous (quadratic lag: $\beta = -.20$, $p < .0001$; Figure 2B) and did not exhibit a clear leader/follower pattern (linear lag: $\beta = .007$, $p = .60$; Figure 2B). As with negativity, the maximum lag for positivity was not significantly different from 0, $t(38) = -1.36$, $p = .18$. Evident by the inverted U shape of the negative quadratic lag (see Figure 2), mother-adolescent dyads demonstrated concurrent synchrony for both negativity and positivity with the highest recurrence rates occurring around simultaneity (i.e., lag of zero), indicating concurrent synchrony in both positive and negative emotional states.

As hypothesized, analyses also revealed a significant effect of the quadratic term in the *reverse* direction (i.e., time-lagged synchrony) for validation/interest states (quadratic lag: $\beta = .08$, $p = .01$, Figure 2C). This suggests that if one party exhibited validation/interest, the other party was less likely to exhibit validation/interest at the same time—supporting our hypothesis that these behaviors should exhibit turn-taking patterns. We see this in the small but evident U-shaped plot in Figure 2C, in contrast to the inverted U shape of positive and negative emotion (Figure 2A and 2B). No significant leader/follower effect was found (linear lag: $\beta = -.02$, $p = .46$), and maximum lag did not significantly differ from 0, $t(33) = -.60$, $p = .56$.

³ Note that slopes are fully nested as random effects as well, so that we "kept it maximal" in the specification of the dyad-level and fixed-factor parts of the model (Barr, 2013). In R syntax, this equation is specified as `lmer(RR_emotion ~ t' + t'' + (1 + fef| dyad), fef)`. *fef* stands for the fixed effects that still converged in the model. See supplementary material for all code used in these analyses.

⁴ In R syntax, this is specified as `lmer(RR_emotion ~ t' + t'' + satisfaction + t' × satisfaction + t'' × satisfaction + (1 + fef| dyad))`. Again, *fef* stands for the fixed effects (now with the interaction term) that still converged in the model. See supplementary material for all code used in these analyses.

⁵ Though we should interpret the standard scales with caution, it is nevertheless useful to consider them on a standard scale. An anonymous reviewer helpfully pointed out that there is no consensus definition of these effect sizes when working with complex models of the kind estimated using `lmer`. In addition, `lmer` and other multi-level model approaches introduce variance/co-variance components and other factors that permit potential investigation at the dyad level. See Mirman (2014) for an elegant introduction to this matter.

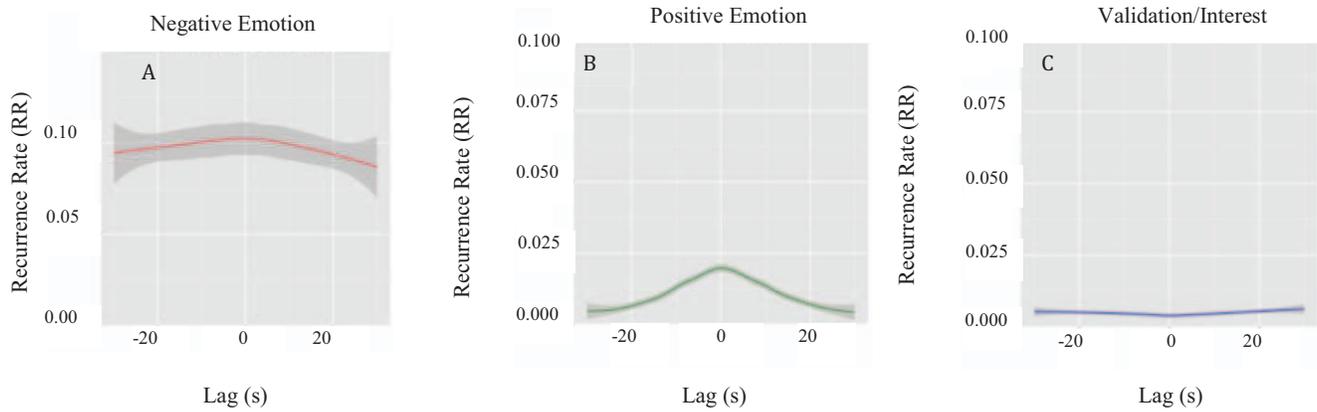


Figure 2. Diagonal recurrence profiles (DRPs) of emotional synchrony across all dyads. For negativity (A) and positivity (B), we see a concurrently synchronous pattern of emotion, with higher recurrence near lag 0, although positive synchrony is considerably lower than negativity in base rate. Validation/interest, however, shows a time-lagged synchronous pattern that can be evidence of turntaking (C). The colored (dark) lines represent the mean *RR*, and the shaded areas represent standard errors. See the online article for the color version of this figure.

Emotion Dynamics Across Discussion Satisfaction

We next examined the relation between dyad-level discussion satisfaction and emotion dynamics across mothers and adolescents with the model presented in Equation 2. The following models include the dyad-level discussion satisfaction (i.e., mean of adolescent and mother-reported discussion satisfaction) as an additional predictor. All model results are included in Table 1. Although all analyses were performed with the continuous satisfaction variable, we present the DRPs across low (i.e., below the median; median = 3.96) and high (i.e., above the median) groups for visualization purposes in Figure 3.

The first model investigated negative emotion synchrony and its relation to discussion satisfaction. As expected, higher discussion

satisfaction predicted lower overall recurrence of negativity (satisfaction: $\beta = -.41$, $p = .002$). The interaction term between quadratic lag and satisfaction, on the other hand, was not significant ($\beta = .02$, $p = .54$). This suggests that despite apparent differences in the DRP, the overall *shape* of the negativity recurrence profile did not differ significantly across levels of discussion satisfaction (see Figure 3A). Overall, less satisfied dyads showed higher levels of concurrently synchronous negativity compared with more satisfied dyads. There was no significant leader/follower pattern (linear lag: $\beta = .01$, $p = .90$).

The second model investigated synchrony of positivity. Despite the visual pattern of higher concurrent synchrony (Figure 3B), high-satisfaction dyads did not show more concurrently synchro-

Table 1
Model Results for Emotion Coordination Across All Three Emotion Types and Discussion Satisfaction

Variable	Negativity	Positivity	Validation/Interest
Satisfaction	$\beta = -.41$ $B = -.06$ $p = .002^{***}$	$\beta = .17$ $B = .004$ $p = .11$	$\beta = .38$ $B = .01$ $p = .002^{**}$
Linear lag	$\beta = .01$ $B = .01$ $p = .90$	$\beta = -.10$ $B = -.02$ $p = .11$	$\beta = .04$ $B = .01$ $p = .74$
Quadratic lag	$\beta = -.05$ $B = -.05$ $p = .06$	$\beta = -.08$ $B = -.05$ $p = .06$	$\beta = -.09$ $B = -.01$ $p = .20$
Satisfaction \times Linear lag interaction	$\beta = -.02$ $B = .01$ $p = .67$	$\beta = -.02$ $B = -.005$ $p = .67$	$\beta = -.07$ $B = -.003$ $p = .61$
Satisfaction \times Quadratic lag interaction	$\beta = .02$ $B = -.005$ $p = .54$	$\beta = .02$ $B = .004$ $p = .54$	$\beta = .17$ $B = .01$ $p = .04^*$

Note. Both standardized (β) and unstandardized (B) coefficients are reported for each model.
* $p < .05$. ** $p < .001$.

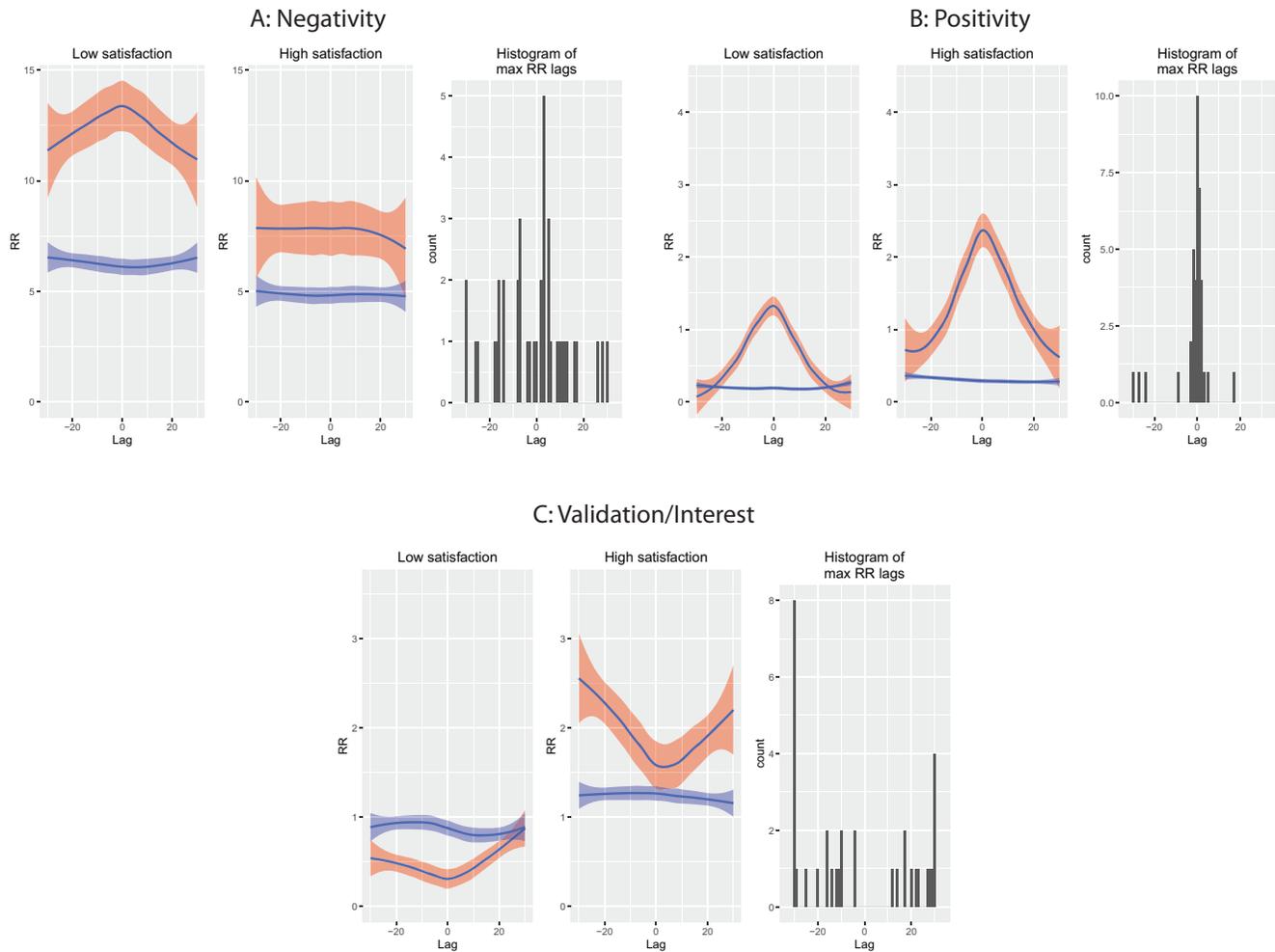


Figure 3. Emotion synchrony between mothers and adolescents across dyadic discussion satisfaction for negativity (A), positivity (B), and validation/interest (C). The red (light) band plots the recurrence of interacting dyads within the target group and emotion; the blue (dark) band plots the shuffled baseline (surrogate analysis). Here for the surrogate analysis we see flat (or non-existent) recurrence between “artificial” dyads. For each band, the blue (dark) line represents the mean RR , and the shaded areas represent standard errors. See the online article for the color version of this figure.

nous positivity than low-satisfaction dyads (satisfaction: $\beta = .17$, $p = .11$). The temporal unfolding of synchrony of positivity did not significantly differ across satisfaction (quadratic lag \times satisfaction: $\beta = .02$, $p = .54$). Positivity also did not demonstrate a clear leader/follower pattern (linear lag: $\beta = -.10$, $p = .06$).

The third model examined synchrony of validation/interest. Increased time-lagged synchrony of validation/interest—characterized by a turn-taking pattern—was reliably predictive of higher discussion satisfaction (satisfaction: $\beta = .38$, $p = .002$). The significant interaction term between quadratic lag and satisfaction ($\beta = .17$, $p = .04$) supports the apparent visual difference in DRP shape across levels of discussion satisfaction. Although both high- and low-satisfaction dyads show the U-shaped pattern of time-lagged synchrony for validation/interest, we see a more highly peaked profile for more satisfied dyads compared with the relatively flatter profile of less satisfied dyads (see Figure 3C). This may be indicative of a stronger

turn-taking dynamic with a shorter delay between individuals’ expressions of validation/interest in dyads that are more satisfied with the discussion. The pattern of results suggests that there were no leader/follower patterns for validation/interest (linear lag: $\beta = .04$, $p = .74$).

Emotion Dynamics Across Adolescent Age

While the previous section paints a picture of emotion dynamics as a function of discussion satisfaction, we were also interested in how emotion dynamics might vary across dyads comprising younger or older adolescents (i.e., 13- to 14-year-olds vs. 17- to 18-year-olds). As mentioned previously, this model was identical to Equation 2 but substituted age for satisfaction. All model results are reported in Table 2. In order to visualize across age, we present the DRPs across younger (i.e., 13- to 14-year-olds) and older (i.e., 17- to 18-year-olds) adolescents in Figure 4, although all models

Table 2
Model Results for Emotion Coordination Across All Three Emotion Types and Adolescent Age

Variable	Negativity	Positivity	Validation/Interest
Adolescent age	$\beta = -.12$ $B = -.01$ $p = .42$	$\beta = -.11$ $B = -.001$ $p = .37$	$\beta = .39$ $B = .005$ $p = .002^{**}$
Linear lag	$\beta = -.16$ $B = -.16$ $p = .03^*$	$\beta = -.07$ $B = -.01$ $p = .50$	$\beta = .28$ $B = .05$ $p = .18$
Quadratic lag	$\beta = -.06$ $B = -.06$ $p = .36$	$\beta = -.35$ $B = -.06$ $p = .34$	$\beta = -.17$ $B = -.03$ $p = .44$
Age \times Linear lag interaction	$\beta = .15$ $B = .01$ $p = .04^*$	$\beta = .08$ $B = .001$ $p = .47$	$\beta = -.31$ $B = -.004$ $p = .16$
Age \times Quadratic lag interaction	$\beta = .03$ $B = .002$ $p = .62$	$\beta = .16$ $B = .002$ $p = .68$	$\beta = .25$ $B = .003$ $p = .27$

Note. Both standardized (β) and unstandardized (B) coefficients are reported for each model.
^{*} $p < .05$. ^{**} $p < .001$.

include the continuous age variable. Although the DRPs for high and low satisfaction models bear strong resemblances to the DRPs for younger and older adolescent models (respectively), we found no significant correlation between age and satisfaction, $t(47) = 1.077$, $p = .29$.

Dyads with younger and older adolescents did not differ significantly in their overall levels of concurrently synchronous negativity (age: $\beta = -.12$, $p = .42$). Interestingly, there was a main effect of linear lag ($\beta = -.16$, $p = .03$), suggesting that there were leader/follower patterns evident in negativity across all ages, as well as a significant interaction between age and linear lag ($\beta = .15$, $p = .04$). Exploration of the plot revealed that mothers led negativity in dyads with younger adolescents and adolescents led negativity in dyads with older adolescents (see Figure 4A). The significant main effect of age in predicting RR of validation/interest ($\beta = .39$, $p = .002$) suggested that dyads with older adolescents had higher levels of recurrent validation/interest than dyads with younger adolescents (see Figure 4C). There were no significant terms in the model for positivity (see Table 2).

Discussion

The present analyses shed new light on questions of emotion dynamics during adolescence, an important period in emotional development and family relationships. To our knowledge, this is the first study to explore dynamics of multiple distinct emotion types in parent–adolescent interactions and to examine their distinct functional consequences for relational outcomes.

Emotion Dynamics Across All Dyads

Our analyses of mother and adolescent emotion dynamics revealed that both negative and positive states generally showed concurrently synchronous patterns (i.e., the maximum amount of synchrony was evident around lag zero). This is consistent with research on emotional flexibility in interpersonal interactions and dynamic systems approaches to emotion (see Butler, 2015; Hol-

enstein, 2015). Validation/interest states showed the opposite pattern, with parties less likely to display validation/interest at the same time. This is evident by the significance of the U-shaped function that indicates a time-lagged synchronous pattern of these emotions between mothers and adolescents. This makes intuitive sense considering the structure of the interaction: One person validates or shows interest in another's feelings while that person shares his or her feelings, creating a turn-taking structure. Adult couples tend to show coordinated emotion patterns during cooperative interactions (Randall et al., 2013), but our study suggests that by adolescence there is already a tendency for adolescents and their parents to engage in positive social coordination.

Analyses of emotion dynamics also revealed that mothers and adolescents generally did not exhibit "leader/follower" patterns across any emotions examined. In other words, the onset of mothers' emotions did not regularly precede the onset of adolescents' emotions or vice versa. These results differ from previous studies of parents and infants, in which parents tend to follow the affective lead of their infants (e.g., Feldman, 2006). Our findings suggest that by the time children reach adolescence, such leader/follower patterns in the parent–child relationship may dissipate (though we discuss age-related differences in leader/follower patterns across adolescence in greater detail below.)

Emotion Dynamics Across Discussion Satisfaction

Our next series of analyses sought to explore whether these patterns reflected perceptions of how conflicts are handled. For negative emotion, the overall level of recurrence of negativity differed as a function of discussion satisfaction: Less satisfied dyads had higher levels of concurrently synchronized (i.e., mutual) negative emotion than did more satisfied dyads. This is consistent with work showing that parent–adolescent interactions that are characterized by high levels of negative emotion are associated with poor adolescent adjustment and parent–adolescent relationship quality (Lougheed, Craig, et al., 2015; Moed et al., 2015). The present study builds on this prior work by showing that not just the

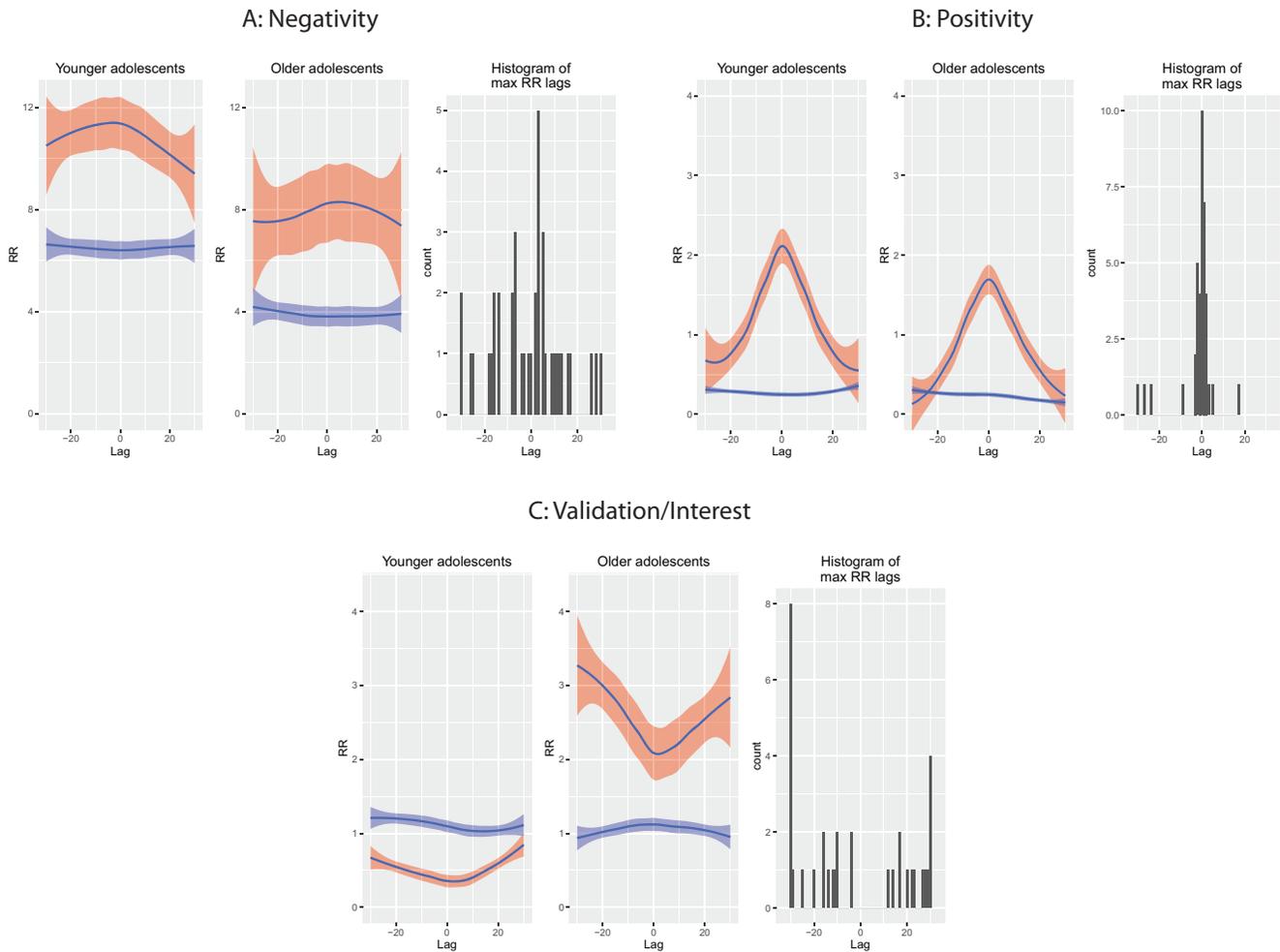


Figure 4. Emotion synchrony between mothers and adolescents across adolescent age for negativity (A) negativity, (B) positivity, and (C) validation/interest. The red (light) band plots the recurrence of interacting dyads within the target group; the blue band plots the shuffled baseline (surrogate analysis). Here for the surrogate analysis we see flat (or non-existent) recurrence between “artificial” dyads. For each band, the blue (dark) line represents the mean *RR*, and the shaded areas represent standard errors. See the online article for the color version of this figure.

overall *amount* of negativity but also the *relative timing* of displayed negative emotions may be particularly damaging to parent–adolescent interactions.

Importantly, although the overall level of recurrence of negativity differed as a function of discussion satisfaction, the *shape* of the recurrence profile of negativity did not. Essentially, when one person expressed negative emotion, the other person was more likely to express negative emotion simultaneously—regardless of how well the conflict was handled. These findings are consistent with research demonstrating that parent–adolescent dyads have a tendency to become “stuck” in negative states (Hollenstein & Lewis, 2006). Our study shows that concurrent synchrony of negative emotion—at least in the context of parent–adolescent conflict—is an affective pattern that cuts across interaction outcomes. However, in light of recent research showing that emotional flexibility between parents and adolescents varies depending on the context (Lougheed & Hollenstein, 2016), future research

could examine patterns of negative synchrony across multiple contexts (e.g., problem-solving, discussing a pleasant topic) and with other populations.

Contrary to our hypotheses, more satisfied dyads did not show significantly higher concurrent synchrony of positive emotion (e.g., humor, affection, enthusiasm) compared with less satisfied dyads. This is inconsistent with previous research showing that positive synchrony during parent–adolescent interactions is linked to better adjustment (Lindsey, Colwell, Fabutt, Chambers, & MacKinnon-Lewis, 2008). This may be partly attributable to the low rate of recurrent positivity across all dyads, likely caused by the conflict context. Future research examining emotional dynamics during whole family interactions (see Hollenstein, Allen, & Sheeber, 2016) is important to illuminate the complexity of emotion dynamics across contexts. This is especially important given the role of the family as a system that is irreducible to any individual within that system (Granic, 2008).

Dyads with higher discussion satisfaction were more likely to show time-lagged synchrony (or turn-taking) of validation/interest than dyads with lower discussion satisfaction. This suggests that dyads' perceived handling of conflict was better if they tended to mutually listen more to one another and validate the others' point of view in a turn-taking structure. This might be indicative of a bidirectional empathic communication process in which dyads that are more willing to empathize with *one another* are better able to resolve conflicts (Halpern, 2007).

Interestingly, dyads with higher discussion satisfaction were also more likely to show a different temporal pattern of time-lagged synchrony of validation/interest states compared with dyads with lower discussion satisfaction. Not only were dyads with higher discussion satisfaction more likely to exhibit validation/interest states overall, but these dyads were also more likely to display validation/interest with stronger turn-taking dynamics and a shorter delay between one another's validation/interest behavior, as seen in the sharper U-shaped curve of their DRPs. In contrast to the tighter turn-taking structure of highly satisfied dyads, dyads with lower satisfaction had lower overall rates of shared validation/interest and a relatively flatter DRP. This suggests that one member of the dyad may validate the other's concern, but their validation—if reciprocated at all—was not reciprocated until up to a minute later. This is consistent with research showing that parents of children with externalizing problems and issues with social competence tend to be less likely to respond with temporally contingent support to their children's emotions compared with parents of typically developing children (Lougheed, Hollenstein, Lichtwarck-Aschoff, & Granic, 2015; Gottman, Katz, & Hooven, 1996). Our results demonstrate the bidirectional nature of validation and interest behaviors in parent–adolescent relationships. Furthermore, the tight temporal dynamics of turn-taking of validation and interest are crucial for successful interactions, not simply the overall amount of validation/interest or even just the fact *that* these behaviors are coordinated in a time-lagged synchronous fashion.

Emotion Dynamics Across Adolescence Age

Our results also shed new light on differences in emotion dynamics across early and late adolescence. Mothers tended to lead negativity in dyads with younger adolescents, and adolescents led negativity in dyads with older adolescents. This is consistent with parent–adolescent relationships becoming more egalitarian from early to late adolescence (de Goede, Branje, & Meeus, 2009) and recent research demonstrating that mothers tend to support their daughters' emotions more over time in dyads with early maturing girls versus late-maturing girls (Lougheed, Hollenstein, & Lewis, 2015).

Furthermore, dyads with older adolescents showed higher levels of recurrent validation/interest than dyads with younger adolescents. Because communication of validation and interest is associated with understanding another's point of view (Halpern, 2007), this finding suggests that this communicative ability improves from early to late adolescence. Indeed, adolescents' empathy and tendency to take others' perspectives increase across adolescence (Eisenberg et al., 2005; van Lissa et al., 2014). Our findings suggest that older adolescents engage in more valida-

tion and interest than younger adolescents, but perhaps more importantly, parents and adolescents in older dyads engage in a more robust *mutual* pattern of time-lagged validation and interest, with overall higher rates of recurrence across all lags. The present study adds to a growing body of literature that parent–adolescent emotion dynamics change over the course of early to late adolescence.

Our findings enrich prior research by demonstrating that *different temporal patterns* of emotions in parent–child interactions—not just the *overall levels* of emotions—are important for understanding intermediate relational outcomes. Indeed, our surrogate analyses (i.e., shuffled baseline of different emotion types within low- and high-satisfaction dyads) serve to adjust for the level of emotion types within each group. The removal of overall levels of each individual's emotional states is important for assessing covariation between individuals (see Butler, 2011). By holding context constant, we are able to more confidently infer that the emotion dynamics of mothers and adolescents emerged in response to one another's emotions and behaviors, rather than a third variable (e.g., an artifact of comparing different conversational contexts). Such emotion dynamics have implications for how well conflicts are handled in the moment, which may compound over time to lead to problematic or more well-functioning parent–adolescent relationships (Granic, 2005).

Limitations and Future Directions

There are some limitations in the present investigation that warrant mentioning. First, the cross-sectional nature of the data limits us from analyzing changes in emotion dynamics across adolescence. Relatedly, we assessed satisfaction with the process and outcome of the conflict discussion immediately following the discussion, which precludes our ability to determine whether emotion dynamics during conflict discussions in the laboratory have implications for long-term conflict management in the home. Future research using longitudinal designs would increase confidence that emotion dynamics are important for long-term conflict outcomes.

Second, we analyzed emotion dynamics across the entire conversation, rather than focusing on distinct periods of exchange, such as starting and ending points of the interactions. An important outstanding question is whether the temporal structure here reflects initiation, completion, or perhaps intermediate synchrony during the interaction. One possibility for future research may lie in using windowed RQA, allowing the researcher to slide a window of a given size over the interaction and examine the structure as it unfolds (Coco & Dale, 2014). Furthermore, we explored synchrony across the *same* emotional state between mothers and adolescents, but an important future direction is to examine synchrony across *different* emotions.

Third, we justified our parameter settings in the Analysis Plan, but there is debate in the literature about such settings. Some have argued that the choice of these parameters for categorical RQA should be guided by an understanding of the system under study. In the present study, we used RQA as a kind of nominal lag sequential analysis that allows wide ranging dynamic exploration of the time series (see Bakeman & Quera, 2011). For simplicity and consistency, we used the simplest parameters (single-instance recurrence) by looking only at second-by-second emotion state

matches. Future research may explore whether other parameter settings could uncover additional information about these interactions (e.g., patterns of entrainment or antiphase patterns; see Butner et al., 2014). Furthermore, our modeling strategy does not focus on the important elements of the random effects and how they are distributed (see Chapter 7 of Mirman, 2014, for a useful discussion of how random effects may shed light on individual variability). Future analysis ought to focus on patterns of variability beneath the fixed effects we explore here.

Conclusion

The present study explored moment-to-moment emotion dynamics between parents and adolescents in novel ways using recurrence quantification analysis, a relatively new technique for analyzing behavioral streams and revealing nonlinear patterns in coupled systems. The present study adds to the interpersonal emotion dynamics literature by demonstrating that the dynamics of distinct emotion types—negativity, positivity, and validation/interest—are differentially associated with perceptions of how well conflicts are managed in parent–adolescent interactions. Dynamic, temporally sensitive analyses complement aggregate analyses by affording in-depth explorations of phenomena that may be washed out with longer timescales. Approaching synchrony through a developmental lens, we found that emotion dynamics have unique signatures across emotion types and are associated with meaningful downstream relational outcomes. Examining such dynamics provides a window into understanding specific behaviors in and their implications for positive interactions during this crucial developmental period for emotional development and family relationships.

References

- Abney, D. H., Paxton, A., Dale, R., & Kello, C. T. (2015). Movement dynamics reflect a functional role for weak coupling and role structure in dyadic problem solving. *Cognitive Processing, 16*, 325–332. <http://dx.doi.org/10.1007/s10339-015-0648-2>
- Allen, J. P., Hauser, S. T., Bell, K. L., & O'Connor, T. G. (1994). Longitudinal assessment of autonomy and relatedness in adolescent-family interactions as predictors of adolescent ego development and self-esteem. *Child Development, 65*, 179–194. <http://dx.doi.org/10.2307/1131374>
- Bakeman, R., & Quera, V. (2011). *Sequential analysis and observational methods for the behavioral sciences*. New York, NY: Cambridge University Press. <http://dx.doi.org/10.1017/CBO9781139017343>
- Barr, D. J. (2013). Random effects structure for testing interactions in linear mixed-effects models. *Frontiers in Psychology, 4*, 328. <http://dx.doi.org/10.3389/fpsyg.2013.00328>
- Bates, D., Maechler, M., Bolker, B., & Walker, S. (2015). *lme4: Linear mixed-effects models using Eigen and S4*. R package version 1.1–8, <http://CRAN.R-project.org/package=lme4>
- Butler, E. A. (2011). Temporal interpersonal emotion systems: The “TIES” that form relationships. *Personality and Social Psychology Review, 15*, 367–393. <http://dx.doi.org/10.1177/1088868311411164>
- Butler, E. A. (2015). Interpersonal affect dynamics: It takes two (and time) to tango. *Emotion Review, 7*, 336–341. <http://dx.doi.org/10.1177/1754073915590622>
- Butner, J. E., Berg, C. A., Baucom, B. R., & Wiebe, D. J. (2014). Modeling coordination in multiple simultaneous latent change scores. *Multivariate Behavioral Research, 49*, 554–570. <http://dx.doi.org/10.1080/00273171.2014.934321>
- Campos, J. J., Walle, E. A., Dahl, A., & Main, A. (2011). Reconceptualizing emotion regulation. *Emotion Review, 3*, 26–35. <http://dx.doi.org/10.1177/1754073910380975>
- Coan, J. A., & Gottman, J. M. (2007). The Specific Affect Coding System (SPAFF). In J. A. Coan & J. A. Allen (Eds.), *Handbook of emotion elicitation and assessment* (pp. 267–285). *Series in affective science*. New York, NY: Oxford University Press.
- Coco, M. I., & Dale, R. (2014). Cross-recurrence quantification analysis of categorical and continuous time series: An R package. *Frontiers in Psychology, 5*, 510. <http://dx.doi.org/10.3389/fpsyg.2014.00510>
- Cole, P. M., Teti, L. O., & Zahn-Waxler, C. (2003). Mutual emotion regulation and the stability of conduct problems between preschool and early school age. *Development and Psychopathology, 15*, 1–18. <http://dx.doi.org/10.1017/S0954579403000014>
- Connell, A. M., Hughes-Scalise, A., Klostermann, S., & Azem, T. (2011). Maternal depression and the heart of parenting: Respiratory sinus arrhythmia and affective dynamics during parent–adolescent interactions. *Journal of Family Psychology, 25*, 653–662. <http://dx.doi.org/10.1037/a0025225>
- Dale, R., & Spivey, M. J. (2006). Unraveling the dyad: Using recurrence analysis to explore patterns of syntactic coordination between children and caregivers in conversation. *Language Learning, 56*, 391–430. <http://dx.doi.org/10.1111/j.1467-9922.2006.00372.x>
- Dale, R., Warlaumont, A. S., & Richardson, D. C. (2011). Nominal cross recurrence as a generalized lag sequential analysis for behavioral streams. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering, 21*, 1153–1161. <http://dx.doi.org/10.1142/S0218127411028970>
- De Goede, I. H., Branje, S. J. T., & Meeus, W. H. J. (2009). Developmental changes in adolescents' perceptions of relationships with their parents. *Journal of Youth and Adolescence, 38*, 75–88. <http://dx.doi.org/10.1007/s10964-008-9286-7>
- Delaherche, E., Chetouani, M., Mahdhaoui, A., Saint-Georges, C., Viaux, S., & Cohen, D. (2012). Interpersonal synchrony: A survey of evaluation methods across disciplines. *IEEE Transactions on Affective Computing, 3*, 349–365. <http://dx.doi.org/10.1109/T-AFFC.2012.12>
- Eisenberg, N., Cumberland, A., Guthrie, I. K., Murphy, B. C., & Shepard, S. A. (2005). Age changes in prosocial responding and moral reasoning in adolescence and early adulthood. *Journal of Research on Adolescence, 15*, 235–260. <http://dx.doi.org/10.1111/j.1532-7795.2005.00095.x>
- Feldman, R. (2003). Infant-mother and infant-father synchrony: The coregulation of positive arousal. *Infant Mental Health Journal, 24*, 1–23. <http://dx.doi.org/10.1002/imhj.10041>
- Feldman, R. (2006). From biological rhythms to social rhythms: Physiological precursors of mother-infant synchrony. *Developmental Psychology, 42*, 175–188. <http://dx.doi.org/10.1037/0012-1649.42.1.175>
- Feldman, R. (2007). Parent-infant synchrony: Biological foundations and developmental outcomes. *Current Directions in Psychological Science, 16*, 340–345. <http://dx.doi.org/10.1111/j.1467-8721.2007.00532.x>
- Field, T., Healy, B. T., Goldstein, S., & Guthertz, M. (1990). Behavior-state matching and synchrony in mother-infant interactions of nondepressed versus depressed dyads. *Developmental Psychology, 26*, 7–14. <http://dx.doi.org/10.1037/0012-1649.26.1.7>
- Fusaroli, R., Konvalinka, I., & Wallot, S. (2014). Analyzing social interactions: Promises and challenges of cross recurrence quantification analysis. *Springer Proceedings in Mathematics & Statistics, 103*, 137–155. http://dx.doi.org/10.1007/978-3-319-09531-8_9
- Gottman, J. M., Katz, L. F., & Hooven, C. (1996). Parental meta-emotion philosophy and the emotional life of families: Theoretical models and preliminary data. *Journal of Family Psychology, 10*, 243–268. <http://dx.doi.org/10.1037/0893-3200.10.3.243>

- Granic, I. (2005). Timing is everything: Developmental psychopathology from a dynamic systems perspective. *Developmental Review, 25*, 386–407. <http://dx.doi.org/10.1016/j.dr.2005.10.005>
- Granic, I. (2008). A dynamic systems approach to understanding family and peer relationships: Implications for effective interventions with aggressive youth. In A. Fogel, B. J. King, & S. G. Shanker (Eds.), *Human development in the 21st century: Visionary ideas from systems scientists* (pp. 216–224). New York, NY: Cambridge University Press.
- Halpern, J. (2007). Empathy and patient-physician conflicts. *Journal of General Internal Medicine, 22*, 696–700. <http://dx.doi.org/10.1007/s11606-006-0102-3>
- Hollenstein, T. (2015). This time, it's real: Affective flexibility, time scales, feedback loops, and the regulation of emotion. *Emotion Review, 7*, 308–315.
- Hollenstein, T., Allen, N. B., & Sheeber, L. (2016). Affective patterns in triadic family interactions: Associations with adolescent depression. *Development and Psychopathology, 28*, 85–96.
- Hollenstein, T., & Lewis, M. D. (2006). A state space analysis of emotion and flexibility in parent-child interactions. *Emotion, 6*, 656–662. <http://dx.doi.org/10.1037/1528-3542.6.4.656>
- Keith, T. Z. (2005). *Multiple regression and beyond*. Boston, MA: Pearson Education.
- Konvalinka, I., Xygalatas, D., Bulbulia, J., Schjødt, U., Jegindø, E. M., Wallot, S., . . . Roepstorff, A. (2011). Synchronized arousal between performers and related spectators in a fire-walking ritual. *PNAS Proceedings of the National Academy of Sciences of the United States of America, 108*, 8514–8519. <http://dx.doi.org/10.1073/pnas.1016955108>
- Kuppens, P. (2015). It's about time: A special section on affect dynamics. *Emotion Review, 7*, 297–300. <http://dx.doi.org/10.1177/1754073915590947>
- Laursen, B., & Collins, W. A. (2004). Parent-child communication during adolescence. In A. L. Vangelisti (Ed.), *Handbook of family communication* (pp. 333–348). Mahwah, NJ: Erlbaum Publishers.
- Laursen, B., Coy, K. C., & Collins, W. A. (1998). Reconsidering changes in parent-child conflict across adolescence: A meta-analysis. *Child Development, 69*, 817–832. <http://dx.doi.org/10.1111/j.1467-8624.1998.00817.x>
- Lichtwarck-Aschoff, A., Hasselman, F., Cox, R., Pepler, D., & Granic, I. (2012). A characteristic destabilization profile in parent-child interactions associated with treatment efficacy for aggressive children. *Nonlinear Dynamics, Psychology, and Life Sciences, 16*, 353–379.
- Lichtwarck-Aschoff, A., Kunnen, S. E., & van Geert, P. L. (2009). Here we go again: A dynamic systems perspective on emotional rigidity across parent-adolescent conflicts. *Developmental Psychology, 45*, 1364–1375. <http://dx.doi.org/10.1037/a0016713>
- Lichtwarck-Aschoff, A., Kunnen, S., & van Geert, P. (2010). Adolescent girls' perceptions of daily conflicts with their mothers: Within-conflict sequences and their relationship to autonomy. *Journal of Adolescent Research, 25*, 527–556. <http://dx.doi.org/10.1177/0743558410367953>
- Lindsey, E. W., & Caldera, Y. M. (2014). Shared affect and dyadic synchrony among secure and insecure parent-toddler dyads. *Infant and Child Development, 24*, 394–413. <http://dx.doi.org/10.1002/icd.1893>
- Lindsey, E. W., Colwell, M. J., Fabutt, J. M., Chambers, J. C., & MacKinnon-Lewis, C. (2008). Mother-child dyadic synchrony in European American and African American families during adolescence: Relations with self-esteem and prosocial behavior. *Merrill-Palmer Quarterly, 54*, 289–315. <http://dx.doi.org/10.1353/mpq.0.0003>
- Lougheed, J. P., Craig, W. M., Pepler, D., Connolly, A. O., Granic, I., & Hollenstein, T. (2015). Maternal and peer regulation of adolescent emotion: Associations with depressive symptoms. *Journal of Abnormal Child Psychology*. Advance online publication. <http://dx.doi.org/10.1007/s10802-015-0084-x>
- Lougheed, J. P., & Hollenstein, T. (2016). Socioemotional flexibility in mother-daughter dyads: Riding the emotional rollercoaster across positive and negative contexts. *Emotion*. Advance online publication. <http://dx.doi.org/10.1037/emo0000155>
- Lougheed, J. P., Hollenstein, T., & Lewis, M. D. (2015). Maternal regulation of daughters' emotion during conflicts from early to mid-adolescence. *Journal of Research on Adolescence*. Advance online publication. <http://dx.doi.org/10.1111/jora.12211>
- Lougheed, J. P., Hollenstein, T., Lichtwarck-Aschoff, A., & Granic, I. (2015). Maternal regulation of child affect in externalizing and typically-developing children. *Journal of Family Psychology, 29*, 10–19. <http://dx.doi.org/10.1037/a0038429>
- Maccoby, E. E., & Martin, J. A. (1983). Socialization in the context of the family: Parent-child interaction. In P. H. Mussen (Series Ed.) and E. M. Hetherington (Vol. Ed.), *Handbook of child psychology: Vol. 4. Socialization, personality, and social development* (4th ed., pp. 1–101). New York, NY: Wiley.
- Marwan, N. (2008). A historical review of recurrence plots. *The European Physical Journal Special Topics, 164*, 3–12. <http://dx.doi.org/10.1140/epjst/e2008-00829-1>
- Mirman, D. (2014). *Growth Curve Analysis and Visualization Using R*. Boca Raton, FL: Chapman & Hall/CRC Press.
- Mirman, D., Dixon, J. A., & Magnuson, J. S. (2008). Statistical and computational models of the visual world paradigm: Growth curves and individual differences. *Journal of Memory and Language, 59*, 475–494. <http://dx.doi.org/10.1016/j.jml.2007.11.006>
- Moed, A., Gershoff, E. T., Eisenberg, N., Hofer, C., Losoya, S., Spinrad, T. L., & Liew, J. (2015). Parent-adolescent conflict as sequences of reciprocal negative emotion: Links with conflict resolution and adolescents' behavior problems. *Journal of Youth and Adolescence, 44*, 1607–1622. <http://dx.doi.org/10.1007/s10964-014-0209-5>
- Netzer, L., Van Kleef, G. A., & Tamir, M. (2015). Interpersonal instrumental emotion regulation. *Journal of Experimental Social Psychology, 58*, 124–135. <http://dx.doi.org/10.1016/j.jesp.2015.01.006>
- Patterson, G. R. (1982). *Coercive family processes*. Eugene, OR: Castalia.
- Paxton, A., & Dale, R. (2013). Frame-differencing methods for measuring bodily synchrony in conversation. *Behavior Research Methods, 45*, 329–343. <http://dx.doi.org/10.3758/s13428-012-0249-2>
- Prinz, R. J., Foster, S., Kent, R. N., & O'Leary, K. D. (1979). Multivariate assessment of conflict in distressed and nondistressed mother-adolescent dyads. *Journal of Applied Behavior Analysis, 12*, 691–700. <http://dx.doi.org/10.1901/jaba.1979.12-691>
- Randall, A. K., Post, J. H., Reed, R. G., & Butler, E. A. (2013). Cooperating with your romantic partner: Associations with interpersonal emotion coordination. *Journal of Social and Personal Relationships, 30*, 1072–1095. <http://dx.doi.org/10.1177/0265407513481864>
- Richardson, D. C., & Dale, R. (2005). Looking to understand: The coupling between speakers' and listeners' eye movement and its relationship to discourse comprehension. *Cognitive Science, 29*, 1045–1060. http://dx.doi.org/10.1207/s15516709cog0000_29
- Richardson, M. J., Marsh, K. L., & Schmidt, R. C. (2005). Effects of visual and verbal interaction on unintentional interpersonal coordination. *Journal of Experimental Psychology: Human Perception and Performance, 31*, 62–79. <http://dx.doi.org/10.1037/0096-1523.31.1.62>
- Vallacher, R. R., Nowak, A., & Zochowski, M. (2005). Dynamics of social coordination: The synchronization of internal states in close relationships. *Interaction Studies: Social Behaviour and Communication in Biological and Artificial Systems, 6*, 35–52.
- van Lissa, C. J., Hawk, S. T., de Wied, M., Koot, H. M., van Lier, P., & Meeus, W. (2014). The longitudinal interplay of affective and cognitive empathy within and between adolescents and mothers. *Developmental Psychology, 50*, 1219–1225. <http://dx.doi.org/10.1037/a0035050>
- Warlaumont, A. S., Richards, J. A., Gilkerson, J., & Oller, D. K. (2014). A social feedback loop for speech development and its reduction in autism. *Psychological Science, 25*, 1314–1324. <http://dx.doi.org/10.1177/0956797614531023>

Webber, C., Jr., & Zbilut, J. (2005). Recurrence quantification analysis of nonlinear dynamical systems. In M. A. Riley & G. C. Van Orden (Eds.), *Tutorials in contemporary nonlinear methods for the behavioral sciences* (pp. 26–94). Arlington, VA: Digital Publication Available through the National Science Foundation. Retrieved from <http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp>

Zaki, J., & Williams, W. C. (2013). Interpersonal emotion regulation. *Emotion, 13*, 803–810. <http://dx.doi.org/10.1037/a0033839>

Zbilut, J. P., Giuliani, A., & Webber, C. L., Jr. (1998). Detecting deterministic signals in exceptionally noisy environments using cross-

recurrence quantification. *Physics Letters. [Part A]*, *246*, 122–128. [http://dx.doi.org/10.1016/S0375-9601\(98\)00457-5](http://dx.doi.org/10.1016/S0375-9601(98)00457-5)

Received April 20, 2015

Revision received February 11, 2016

Accepted February 15, 2016 ■